Question 83
A Stock costing ₹ 150 pays no dividends. The possible prices at which the stock may be sold for at the end of the year with the respective probabilities are:

Price (in ₹)	Probability
130	0.2
150	0.1
160	0.1
165	0.3
175	0.1
180	0.2
Total	1.0

You are required to:
1. calculate the Expected Return,
2. calculate the Standard Deviation (σ) of Returns.

Show calculations upto three decimal points.

(May 17, 8 Marks)

Solution
Here, the probable returns have to be calculated using the formula

$$R = \frac{D}{P_0} = \frac{P_1 - P_0}{P_0}$$

Calculation of Probable Returns

Possible prices (P_1) ₹	$P_1 - P_0$ ₹	$[(P_1 - P_0)/P_0] \times 100$ Return (per cent)
130	-20	-13.33
150	0	0.00
160	10	6.67
165	15	10.00
175	25	16.667
180	30	20.00

Calculation of Expected Returns

Possible return X_i	Probability $p(X_i)$	Product $X_1 - p(X_i)$
-13.333	0.2	-2.667
0.00	0.1	0.000
6.667	0.1	0.667
10.00	0.3	3.000
16.667	0.1	1.667
20.00	0.2	4.000
		X = 6.667

Expected return X = 6.667 per cent
Alternatively, it can also be calculated as follows:

Expected Price
= 130 X 0.2 + 150 X 0.1 + 160 X 0.1 + 165 X 0.3 + 175 X 0.1 + 180 X 0.2 = 160

Return

$$= \frac{160 - 150}{150} \times 100 = 6.667\%$$

Calculation of Standard Deviation of Returns

Probable return X_i	Probability $p(X_i)$	Deviation $(X_i - \bar{X})$	Deviation squared $(X_i - \bar{X})^2$	Product $(X_i - \bar{X})^2 p(X_i)$
-13.333	0.2	-20.00	400.00	80.00
0.00	0.1	-6.667	44.449	4.445
6.667	0.1	0	0	0
10.00	0.3	3.333	11.109	3.333
16.667	0.1	10.00	100.00	10.00
20.00	0.2	13.333	177.769	35.554
				$\sigma^2 = 133.332$

Variance, $\sigma^2 = 133.332$
Standard deviation,
$\sigma = \sqrt{133.332}$
= 11.547 per cent

Question 84

The following information are available with respect of Krishna Ltd.

Year	Krishna Ltd. Average share price (₹)	Dividend per Share (₹)	Average Market Index	Dividend Yield	Return on Govt. bonds
2012	245	20	2013	4%	7%
2013	253	22	2130	5%	6%
2014	310	25	2350	6%	6%
2015	330	30	2580	7%	6%

Compute Beta Value of the Krishna Ltd. at the end of 2015 and state your observation.

(May 2017, 8 Marks)

Solution

1. **Computation of Beta Value**

   Calculation of Returns

$$\text{Returns} = \frac{D_1 + (P_1 - P_0)}{P_0} \times 100$$

Year 2012 – 2013

$$= \frac{22 + (253 - 245)}{245} \times 100 = 12.24\%$$

Return 12.24%

Year 2013 – 2014

$$= \frac{25 + (310 - 253)}{253} \times 100 = 32.14\%$$

Return 32.14%

Year 2014 – 2015

$$= \frac{30 + (330 - 310)}{310} \times 100 = 16.13\%$$

Return 16.13%

Calculation of Returns from market Index

Year	% of Index Appreciation	Dividend Yield %	Total Return %
2012 – 2013	$\frac{(2130 - 2013)}{2013} \times 100 = 5.81\%$	5%	10.81%
2013 – 2014	$\frac{(2350 - 2130)}{2130} \times 100 = 10.33\%$	6%	16.33%
2014 – 2015	$\frac{(2580 - 2350)}{2350} \times 100 = 9.79\%$	7%	16.79%

Computation of Beta

Year	Krishna Ltd. (X)	Market Index (Y)	XY	Y2
2012–13	12.24%	10.81%	132.31	116.86
2013–14	32.41%	16.33%	529.25	266.67
2014–15	16.13%	16.79%	270.82	281.90
Total	60.78%	43.93%	932.38	665.43

$$\text{Average Return of Krishna Ltd.} = \frac{60.78}{3} = 20.26\%$$

$$\text{Average Market Return} = \frac{43.93}{3} = 4.64\%$$

$$\text{Beta } (\beta) = \frac{\sum XY - nX\,Y}{\sum Y^2 - n(Y)^2} = \frac{932.38 - 3 \times 20.26 \times 14.64}{665.43 - 3(14.64)^2} = 1.897$$

2. Observation

	Expected Return (%)	Actual Return (%)	Action
2012 – 13	6%+ 1.897(10.81% - 6%) = 15.12%	12.24%	Sell
2013 – 14	6%+ 1.897(16.33% - 6%) = 25.60%	32.41%	Buy
2014 – 15	6%+ 1.897(16.79% - 6%) = 26.47%	16.13%	Sell

Question 85

The five portfolios of a mutual fund experienced following result during last 10 years periods:

Portfolio	Average annual return %	Standard Deviation	Correlation with the market return
A	20.0	2.3	0.8869
B	17.0 ·	1.8	0.6667
C	18.0	1.6	0.600
D	16.0	1.8	0.867
E	13.5	1.9	0.5437

Market risk : 1.2
Market rate of return : 14.3%
Risk free rate : 10.1%

Beta may be calculated only upto two decimal. Rank the portfolio using JENSEN'S ALPHA method.

(May 17, 8 Marks)

Solution

Let portfolio standard deviation be σ_p
Market Standard Deviation $= \sigma_m$
Coefficient of correlation $= r$

$$\text{Portfolio beta } (\beta_p) = \frac{\sigma_p r}{\sigma_m}$$

$$\text{Beta for A} = \frac{2.30 \times 0.8869}{1.2} = 1.7$$

Required portfolio return (Rp) = Rf + βp (Rm – Rf),
[Rp for A = 10.1 +1.70 X (14.3 – 10.1) = 17.24, etc.]

Portfolio	**Beta**	**Return from the portfolio (Rp) (%)**
A	1.70	17.24
B	1.00	14.30
C	0.80	13.46
D	1.30	15.56
E	0.86	13.71

Portfolio	**Actual Return %**	**Expected Return %**	**Jensen's Alpha**	
			AR – ER	**Rank**
A	20	17.24	2.76	II
B	17	14.30	2.70	III
C	18	13.46	4.54	I
D	16	15.56	0.44	IV
E	13.5	13.71	-0.21	V

Question 86

The return of security 'L' and security 'K' for the past five years are given below:

Year	Security-L Return %	Security-K Return %
2012	10	11
2013	04	– 06
2014	05	13
2015	11	08
2016	15	14

Calculate the risk and return of portfolio consisting above information.

(Nov 17, 10 Marks)

Solution 86

Year	L %	K %	$d_L = L - \bar{L}$	$d_K = K - \bar{K}$	$d_L^{\ 2}$	d_k^2	$d_L.d_k$
2012	10	11	1	3	1	9	3
2013	4	-6	-5	-14	25	196	70
2014	5	13	-4	5	16	25	-20
2015	11	8	2	0	4	0	0
2016	15	14	6	6	36	36	36
	$\sum L$ =45	$\sum k$ = 40			$\sum d_L^2$ = 82	$\sum d_K^2$ = 266	$\sum d_L.d_K$ = 89

WN1: Average Rate of Return:

$$\bar{L} = \frac{\sum L}{N} = \frac{45}{5} = 9\ \%$$

$$\bar{k} = \frac{\sum L}{N} = \frac{40}{5} = 8\ \%$$

WN2: Variance and Standard deviation:

$$\sigma_L^2 = \frac{\sum d_L^2}{N} = \frac{82}{5} = 164$$

$\sigma_L = \sqrt{\sigma_L^2} = \sqrt{16.4} = 4.05\%$

$$\sigma_k^2 = \frac{\sum d_k^2}{N} = \frac{266}{5} = 53.2$$

$\sigma_k = \sqrt{\sigma_k^2} = \sqrt{53.2} = 72.9\%$

WN3: Covariance:

$$Cov_{Lk}^2 = \frac{\sum d_L d_k}{N} = \frac{89}{5} = 17.8$$

Assuming 50% investment in each of the two securities:

1. Return on Portfolio

   $R_p = R_L.W_L + R_K.W_K$

   $R_p = (9)\ (0.5) + (8)\ (0.5) = 8.5\%$

2. Portfolio Risk

   $\sigma_p = \sqrt{\sigma_L^2.\ W_L^2 + \sigma_k^2.\ W_k^2 + 2Cov_{Lk}.\ W_L.\ W_K}$

   $\sigma_p = \sqrt{(16.4)\ (0.5)^2 + (53.2)\ (0.5)^2 + 2\ (17.8)\ (0.5)\ (0.5)}$

   $\sigma_p = \sqrt{26.3} = 5.13\%$

Question 87

Consider the following information on two stocks, X and Y.

Year	2016	2017
Return on X (%)	10	16
Return on Y (%)	12	18

You are required to calculate:

1. The expected return on a portfolio containing X and Y in the proportion of 40% and 60% respectively.
2. The Standard Deviation of return from each of the two stocks.
3. The Covariance of returns from the two stocks.
4. The Correlation coefficient between the returns of the two stocks.
5. The risk of a portfolio containing X and Y in the proportion of 40% and 60%.

(May 18, 8 Marks)

Solution

1. Expected return of portfolio containing X and Y in the ratio 40%,60%

$$E(X) = \frac{(10+16)}{2} = 13\%$$

$$E(Y) = \frac{(12+18)}{2} = 15\%$$

$$R_p = \sum_{I-1}^{N} X_i \; R_i = 0.4(13) + 0.6(15) = 14.2\%$$

2. Standard Deviation of X and Y

Stock X:

Variance = $0.5\,(10 - 13)^2 + 0.5\,(16 - 13)^2 = 9$

Standard deviation = 3%

Stock Y:

Variance = $0.5\,(12 - 15)^2 + 0.5\,(18 - 15)^2 = 9$

Standard deviation = 3%

3. $Cov_{XY} = 0.5\,(10 - 13)\,(12 - 15) + 0.5\,(16 - 13)\,(18 - 15) = 9$

4. Correlation Coefficient = p = $\dfrac{Cov\,(X, Y)}{\sigma x \sigma y} = \dfrac{9}{9} = 1$

5. Risk of portfolio containing 40% X and 60 % Y

$$\sigma p = \sqrt{w_X^2\sigma_X^2 + w_Y^2\sigma_Y^2 + 2w_Xw_Y\,(\sigma_X\sigma_Y Cor._{XY})}$$

$$= \sqrt{(0.4)^2(3)^2 + (0.6)^2(3)^2 + 2(0.4)(0.6)(3)(3)(1)}$$

$$= \sqrt{1.44 + 3.24 + 4.32} = 3\%$$

Question 88

Mr. Kapoor owns a portfolio with the following characteristics:

	Security X	Security Y	Risk Free Security
Factor 1 sensitivity	0.75	1.50	0
Factor 2 sensitivity	0.60	1.10	0
Expected Return	15%	20%	10%

It is assumed that security returns are generated by a two factors model.

1. If Mr. Kapoor has ₹ 1,00,000 to invest and sells short ₹ 50,000 of security Y and purchases ₹ 1,50,000 of security X, what is the sensitivity of Mr. Kapoor's portfolio to the two factors?
2. If Mr. Kapoor borrows ₹ 1,00,000 at the risk free rate and invests the amount he borrows along with the original amount of ₹ 1,00,000 in security X and Y in the same proportion as described in part (i), what is the sensitivity of the portfolio to the two factors?
3. What is the expected return premium of factor 2?

(Nov 18, 8 Marks)

Solution

1. **Mr. Kapoor's position in the two securities is +1.50 in security X and -0.5 in security Y. Hence the portfolio sensitivities to the two factors:**
   b prop. 1 = 1.50 X 0.75 + (-0.50 X 1.50) = 0.375
   b prop. 2 = 1.50 X 0.60 + (-0.50 X 1.10) = 0.35

2. **Mr. Kapoor's current position:**

Security X	₹ 3,00,000 / ₹ 1,00,000	= 3
Security Y	-₹ 1,00,000 / ₹ 1,00,000	= -1
Risk free asset	-₹ 100000 / ₹ 100000	= -1

   b prop. 1 = 3.0 X 0.75 + (-1 X 1.50) + (- 1 X 0) = 0.75
   b prop. 2 = 3.0 X 0.60 + (-1 X 1.10) + (-1 X 0) = 0.70

3. **Expected Return = Risk Free Rate of Return + Risk Premium**
   Let λ_1 and λ_2 are the Value Factor 1 and Factor 2 respectively.
   Accordingly
   $15 = 10 + 0.75\ \lambda_1 + 0.60\ \lambda_2$
   $20 = 10 + 1.50\ \lambda_1 + 1.10\ \lambda_2$
   On solving equation, the value of λ_1 and λ_2 comes 6.67 and 0 respectively.

   Accordingly, the expected risk premium for the factor 2 shall be Zero and whatever be the risk the same shall be on account of factor 1.

   Alternatively, the risk premium of Securities X & Y can be calculated as follows:

   <u>**Security X**</u>
   Total Return = 15%
   Risk Free Return = 10%
   Risk Premium = 5%

Security Y

Total Return = 20%

Risk Free Return = 10%

Risk Premium = 10%

Question 89

Following are the details of a portfolio consisting of 3 shares:

Shares	Portfolio Weight	Beta	Expected Return (%)	Total Variance
X Ltd.	0.3	0.50	15	0.020
Y Ltd.	0.5	0.60	16	0.010
Z Ltd.	0.2	1.20	20	0.120

Standard Deviation of Market Portfolio Return = 12%

You are required to calculate the following:

1. The Portfolio Beta.
2. Residual Variance of each of the three shares.
3. Portfolio Variance using Sharpe Index Model.

(May 19, 8 Marks)

Solution

1. **Portfolio Beta**

0.30 X 0.50 + 0.50 X 0.60 + 0.20 X 1.20
= 0.15 + 0.3 + 0.24 = 0.69

2. **Residual Variance**

To determine Residual Variance first of all we shall compute the Systematic Risk as follows:

$\beta_X^2 \text{ X } \sigma_M^2 = (0.5)^2(0.12)^2 = 0.0036$

$\beta_Y^2 \text{ X } \sigma_M^2 = (0.6)^2(0.12)^2 = 0.0052$

$\beta_Z^2 \text{ X } \sigma_M^2 = (1.20)^2(0.12)^2 = 0.0207$

Residual Variance =	Total Variance	–	Systematic Risk
X =	0.020 – 0.0036	=	0.0164
Y =	0.010 – 0.0052	=	0.0048
Z =	0.120 – 0.0207	=	0.0993

3. **Portfolio variance using Sharpe Index Model**

Portfolio Variance	=	Systematic Risk of the Portfolio	+	Unsystematic Risk of the
Systematic Variance of Portfolio	=	$(0.12)^2 \text{ X } (0.69)^2$	=	0.006856
Unsystematic Variance of Portfolio	=	$0.0164 \text{ X } (0.30)^2 + 0.0048 \text{ X } (0.50)^2 + 0.0993 \text{ X } (0.20)^2$	=	0.006648
Total Variance	=	0.006856+ 0.006648	=	0.013504

Portfolio Variance = Systematic Risk of the Portfolio + Unsystematic Risk of the

$= (0.12)^2 \times (0.69)^2 + 0.0164 \times (0.30)^2 + 0.0048 \times (0.50)^2 + 0.0993 \times (0.20)^2$
$= 0.006856 + 0.006648 = 0.013504$

Question 90
Mr. X holds the following portfolio:

Securities	Cost (₹)	Dividend (₹)	Market Price (₹)	Beta
Equity Shares:				
A Ltd.	16,000	1,600	16,400	0.9
B Ltd.	20,000	1,600	21,000	0.8
C Ltd.	32,000	1,600	44,000	0.6
PSU Bods	68,000	6,800	64,600	0.4

The risk-free rate of return is 12%

Calculation the following:
1. The expected rate of return on his portfolio using Capital Asset Pricing Model (CAPM).
2. The average return on is portfolio. (Calculation up to two decimal points)

(Nov 19, 8 Marks)

Solution
Calculation of expected return on market portfolio (R_m)

Investment	Cost (₹)	Dividends (₹)	Capital Gains (₹)
Shares A	16,000	1,600	400
Shares B	20,000	1,600	1,000
Shares C	32,000	1,600	12,000
PSU Bonds	68,000	6,800	–3,400
	1,36,000	**11,600**	**10,000**

$$R_m = \frac{11,600 + 10,000}{1,36,000} \times 100 = 15.88\%$$

Calculation of expected rate of return on individual security:

Shares A	12 + 0.9 (15.88 – 12.0)	= 15.49%
Shares B	12 + 0.8 (15.88 – 12.0)	= 15.10%
Shares C	12 + 0.6 (15.88 – 12.0)	= 14.33%
PSU Bonds	12 + 0.4 (15.88 – 12.0)	= 13.55%

Calculation of the Average Return of the Portfolio:

$$= \frac{15.49 + 15.10 + 14.33 + 13.55}{4} = 14.62\%$$

Question 91

The risk free rate of return is 5%. The expected rate of return on the market portfolio is 11%. The expected rate of growth in dividend of X Ltd. is 8%. The last dividend paid was ₹ 2.00 per share. The beta of X Ltd. equity stock is 1.5.

i. What is the present price of the equity stock of X Ltd.?

ii. How would the price change when?
- The inflation premium increases by 3%
- The expected growth rate decreases by 3% and
- The beta decreases to 1.3.

(May 18, 4 Marks)

Solution

i. Equilibrium price of Equity using CAPM

$$E(R) = R_f + (R_m - R_f)\beta$$
$$= 5\% + 1.5\ (11\% - 5\%)$$
$$= 5\% + 9\% = 14\%$$

$$P_0 = \frac{D_1}{K_e - g} = \frac{2.00\ (1.08)}{0.14 - 0.08} = \frac{2.16}{0.06} = ₹\ 36$$

ii. New Equilibrium price of Equity using CAPM (assuming 3% on 5% is inflation increase)

$$E(R) = R_f + (R_m - R_f)\beta$$
$$= 5.15\% + 1.3\ (11\% - 5.15\%)$$
$$= 5.15\% + 7.61\% = 12.76\%$$

$$P_0 = \frac{D_1}{K_e - g} = \frac{2.00\ (1.05)}{0.1276 - 0.05} = ₹\ 27.06$$

Alternatively, it can also be computed as follows, assuming it is 3% in addition to 5%

$$= 8\% + 1.3\ (11\% - 8\%)$$
$$= 8\% + 3.9\% = 11.9\%$$

$$P_0 = \frac{D_1}{K_e - g} = \frac{2.00\ (1.05)}{0.119 - 0.05} = ₹\ 30.43$$

Alternatively, if all the factors are taken separately then solution of this part will be as follows:

i. **Inflation Premium increase by 3%.**

This raises RX to 17%. Hence, new equilibrium price will be:

$$= \frac{2.00\ (1.08)}{0.17 - 0.08} = ₹\ 24$$

ii. **Expected Growth rate decrease by 3%.**

Hence, revised growth rate stand at 5%:

$$= \frac{2.00\ (1.05)}{0.14 - 0.05} = ₹\ 23.33$$

iii. Hence, revised cost of equity shall be:

= 5% + 1.3 (11% – 5%)

= 5% + 7.8% = 12.8%

As a result, New Equilibrium price shall be

$$P_0 = \frac{D_1}{K_e - g} = \frac{2.00\ (1.08)}{0.128 - 0.08} = ₹\ 45$$

Question 92

Following are risk and return estimates for two stocks:

Stock	Expected returns (%)	Beta	Specific SD of expected return (%)
A	14	0.8	35
B	18	1.2	45

The market index has a Standard Deviation (SD) of 25% and risk-free rate on treasury Bill is 6%.

You are required to calculate:

i. The standard deviation of expected returns on A and B.

ii. Suppose a portfolio is to be constructed with the proportions of 25%, 40% and 35% in stock A, B and treasury bills respectively, what would be the expected return, standard deviation of expected return of the portfolio?

(Nov 19, 8 Marks)

Solution

i. Total Risk = Systematic Risk + Unsystematic Risk

Unsystematic Risk = 45^2

Total Risk

= σ_b

= $\sqrt{900 + (35)^2}$

= $\sqrt{2{,}925}$

= 54.08%

ii. Expected return of the portfolio

= (0.25 X 14) + (0.40 X 18) + (0.35 X 6)

= 12.8%

Total Risk = Systematic Risk + Unsystematic Risk

Systematic Risk $\beta p^2 \sigma^2_m$

βp = 0.25 (0.8) + 0.4 (1.2) + 0.35 (0)

= 0.2 + 0.48 + 0

= 0.68

Systematic Risk of Portfolio

= $\sqrt{(0.68)^2 + (25)^2}$

= $\sqrt{289}$

Non-systematic Risk of Portfolio

$= (0.25)^2 (35)^2 + (0.40)^2 (45)^2 + 0$

$= 76.56 + 324$

$= \sqrt{400.56}$

Total Risk

$= \sqrt{289 + 400.56}$

$= 26.26$